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IMPROVING CRITICAL THINKING AND INFERENCE SKILLS WITH THE MONTY HALL PROBLEM

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ABSTRACT

Students’ critical thinking skills have been linked to success in undergraduate auditing classes (Jenkins, 1998), and to their perceived success in public accounting (Baril et al., 1998). This paper presents an assignment used in an undergraduate auditing class that serves as a steppingstone in the development of critical thinking, technology, and communication skills. The assignment requires students to solve a problem whose answer is not intuitive to many people and requires the use of spreadsheets and written communication. It should be noted that this exercise has general applicability across multiple business disciplines.

I. INTRODUCTION

The Accounting Education Change Commission (AECC, 1990) proclaims the formal education of an accounting student should lay the foundation on which life-long learning can be built. “Students should be taught ‘how to learn.’ To learn, one must be able to order and structure the facts and data observed. Thus, critical thinking is essential to ‘learning how to learn.’” (Baril et al. 1998, p. 386)

One aspect of learning how to learn is developing skills for making choices under uncertainty. Auditing is one area of accounting where making judgments under uncertainty is a critical skill. “Auditing is essentially an inferential practice and auditors must collect and analyze evidential material in order to form conclusions. [Verification] and evidence are complementary concepts; auditors verify on the basis of evidence.” (Power 1997, p.69)

Inferences within auditing often must rely on subjective probabilities. Kahneman and Tversky (1972) provide an example where people often fail to make the correct choice under uncertainty. In the “maternity ward” problem, students were told a given town is served by two hospitals – a large hospital (where about 45 babies are born daily) and a small hospital (where about 15 babies are born daily). The students were reminded that about 50% of all babies are boys, and the exact mix of
male and female babies on any given day varies. Students were then asked: If, over a one-year period, each hospital recorded the number of days where more than 60% of the babies born were male, which hospital would have more such days? Answer choices are: (1) the large hospital, (2) the small hospital, or (3) about the same. The majority of subjects responded, “about the same.” The correct answer is the small hospital is more likely to have such a birth rate on any given day.

Developing critical thinking, technology, and communication skills is a continuous task. Research has shown students’ critical thinking skills have been linked to success in undergraduate auditing classes (Jenkins, 1998), and to their perceived success in public accounting (Baril et al., 1998). The purpose of this paper is to provide one means of addressing the need for improved levels of these skills for accounting students. The paper presents an assignment used in an undergraduate auditing class that serves as a steppingstone in the development of these skills. The assignment requires students to solve a problem whose answer is not intuitive to many people and requires the use of spreadsheets and written communication.

II. THE PROBLEM

Oh, The Games People Play...

You are the Director of Internal Audit for a major corporation. Company headquarters has requested a special audit that needs to be completed as soon as possible. Financial personnel in the CFO’s office have noted discrepancies at one of the divisions and are questioning the operating results of the division. The CFO believes the Division Director is responsible for the discrepancies.

The division has 3 operating locations (D1, D2, and D3), and the CFO’s office is confident the vast majority of the issues have occurred at one operating location. At this point, the CFO does not know which operating location is the site of the alleged fraud.

You have three people available for the audit – two staff members and one intern. Noting your staffing constraints, the CFO wants a thorough audit at one location and a summary audit at another location. You decide to send a staff member and the intern to the thorough audit location, and one staff member to the summary audit location.

You meet with the Division Director and discuss the impending audits. You tell the Division Director that the CFO will randomly select the location for a summary audit. The Division Director will then have the opportunity to choose the location of
the thorough audit. Finally, you reserve the right to change the location of the summary audit after the Division Director selects the site for the thorough audit.

Suppose the CFO selects D3 for the summary audit and the Division Director selects D2 for the thorough audit (it does not matter which location is originally chosen by the CFO—recall the CFO is making a random choice). Knowing the Division Director has the incentive to avoid being caught (and therefore will never choose the operating location where the alleged fraud exists), should you perform the summary audit at D3 or switch the summary audit to D1?

You know your company has the reputation of completing similar arrangements, as agreed upon, to maintain credibility with management. In other words, all parties are confident that once the Division Director chooses a site for the thorough audit, that location will remain the site for the thorough audit; the company will not change the rules and audit the two locations not chosen by the Division Director. Finally, let’s assume the following: If the location selected by the CFO for the summary audit is the site of the alleged fraud, the Division Director randomly chooses between the other two locations for the thorough audit (call this Assumption 1).

Required: Once the Division Director selects the site for the thorough audit, should you switch the location of the summary audit? For example, if the CFO selects D3 for the summary audit and the Division Director selects D2 for the thorough audit, should you perform the summary audit at D3 or D1?

You must provide a convincing argument for your choice. Your argument should use standard probability rules (potentially including conditional probabilities and Bayes’ Theorem). Some of you will find it helpful to draw a probability tree, or to generate a table of intersection and marginal probabilities. Others may wish to create a more standard probability argument.

In addition to your formal argument, you must provide supporting empirical evidence for your argument. A simulation of the problem can be presented using any number of methods—one method must be to model this scenario using Microsoft Excel. You must submit your answer electronically. The file should contain a Word document and at least one Excel spreadsheet.

III. WHY THIS PROBLEM WORKS

This problem has been a useful teaching tool because it presents an intrinsically interesting question, involves constraints, uncertainty, and a real-world
issue. Moreover, it requires that students think critically, use technology, and communicate in an effective manner.

Many people jump intuitively to the answer that once the Division Director chooses the site for the thorough audit, it does not matter if you switch the location of the summary audit or not – the probability of finding the fraud is 50-50. This error is similar to the error people make in the “maternity ward” problem. As is the case with many good problems, people’s first thoughts do not always provide the best answer. This is where a simulation of the problem can show its true benefits. Requiring a simulation and a formal argument will encourage students to think their way through the problem, instead of just looking for an easy answer. Some students will struggle to create a formal argument because they cannot find an easy starting point. By simulating the problem, students can gain an understanding of the basis of the problem and a potential answer – in this manner they acquire a plan for their formal argument.

The key to understanding the problem is to note you must deal with conditional probabilities. At the beginning, the CFO has a 1/3 chance of finding the alleged fraud. After the Division Director chooses the location for the thorough audit, the internal audit staff has new, useful information. The internal audit staff now knows one location that does not have fraudulent financial statements. At this point, the internal audit staff faces a conditional probability problem. Given that the Division Director has selected a site without fraud, should the internal audit staff audit the division chosen by the CFO or switch to the remaining division?

IV. SOLUTION

The answer is that the internal audit staff should switch the location of the summary audit. Given Assumption 1, the probability of finding the fraud if you switch locations for the summary audit is 2/3. Even without Assumption 1, the noted probability is ≥ 1/2. Using the assumption allows students more direction when they are developing a simulation.

To see the answer, note the following:
- \( P[\text{fraud at D1}] = 1/3; \ P[\text{fraud at D2}] = 1/3; \ P[\text{fraud at D3}] = 1/3 \)

- Assume the CFO has selected D3 for the summary audit:
  - \( P[\text{Division Director chooses D1 | fraud at D1}] = 0; \)
  - \( P[\text{Division Director chooses D1 | fraud at D2}] = 1; \)
  - \( P[\text{Division Director chooses D1 | fraud at D3}] = 1/2; \)
  - \( P[\text{Division Director chooses D2 | fraud at D1}] = 1; \)
P[Division Director chooses D2 | fraud at D2] = 0;
P[Division Director chooses D2 | fraud at D3] = 1/2;
P[Division Director chooses D3 | anything] = 0;

- The table of intersection and marginal probabilities is

<table>
<thead>
<tr>
<th>Div. Dir. Chooses</th>
<th>Fraud located at</th>
</tr>
</thead>
<tbody>
<tr>
<td>D1</td>
<td>D2</td>
</tr>
<tr>
<td>(0) (1/3)</td>
<td>(1) (1/3)</td>
</tr>
<tr>
<td>(1/3)</td>
<td></td>
</tr>
<tr>
<td>Div. Dir. Chooses</td>
<td>Fraud located at</td>
</tr>
<tr>
<td>D2</td>
<td>D1</td>
</tr>
<tr>
<td>(1) (1/3)</td>
<td>(0) (1/3)</td>
</tr>
<tr>
<td>(1/3)</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>1/3</td>
</tr>
</tbody>
</table>

- We want to calculate the probability of finding the fraud using a summary audit.

1. If the fraud is at D1, the Division Director will choose D2. So the P[fraud is at D3 | Div. Dir. Chose D2] = (1/6) / (0 + 1/3 + 1/6) = 1/3; and the P[fraud is at D1 | Div. Dir. Chose D2] = (1/3) / (0 + 1/3 + 1/6) = 2/3. The internal audit staff should change the location of the summary audit from D3 to D1.

2. If the fraud is at D2, the Division Director will choose D1. So the P[fraud is at D3 | Div. Dir. Chose D1] = (1/6) / (0 + 1/3 + 1/6) = 1/3; and the P[fraud is at D2 | Div. Dir. Chose D1] = (1/3) / (0 + 1/3 + 1/6) = 2/3. The internal audit staff should change the location of the summary audit from D3 to D2.

3. If the fraud is at D3, the Division Director will randomly choose between D1 and D2. In this case the internal audit staff should not switch the location of the summary audit.

We can now see that the internal audit department will find the fraud 2/3 of the time if they switch locations of the summary audit after the Division Director chooses a site for the complete audit.

V. DISCUSSION

The general aspects of this problem are often presented under the names of “The Monty Hall Problem,” “The Car and Goats Problem,” and “Marilyn’s Problem.” More detailed discussions can be found in Barbeau (1993), Gillman (1992), and Morgan, et al. (1991). The beauty of the general problem is that it can be tailored to fit
any college of business discipline. In any scenario, students will need to use logic and typically must consider the problem from multiple viewpoints. Therefore, the use of critical thinking skills is a necessary requirement to complete the assignment. To simulate the problem in Excel, a number of students will have to expand their knowledge of spreadsheet capabilities. Students must also spend adequate time documenting their work and conclusions so that the reader can easily follow their train of logic.

I often hear students state that they do not know where to start. Starting with the simulation allows them to realize that the answer “it does not matter whether the internal audit department switches the location of the summary audit or not” is incorrect. By recalculating values in their spreadsheet, a number of times, students will see that the probability of finding the fraud by switching the location of the summary audit approaches 2/3. Now, they are well on their way to critically thinking about the problem. The final hurdle is for the students to succinctly explain their logic for their solution. This provides an excellent opportunity for students to display their communication skills. Instructors can help students build their critical thinking and communication skills by offering critiques of their thought processes and written communication and allowing students to resubmit the assignment. Alternatively, instructors can assign other similar assignments so students can incorporate the instructor’s suggestions.

I offer the following items from the Foundation for Critical Thinking (1997) to my students to help them assess elements of their critical thinking as they consider the problem:

1. All reasoning is based on data, information, and evidence.
   - Restrict your claims to those supported by the data you have.
   - Search for information that opposes your position as well as information that supports it.
   - Make sure that all information used is clear, accurate, and relevant to the question at issue.
   - Make sure you have gathered sufficient information.

2. All reasoning contains inferences or interpretations by which we draw conclusions and give meaning to data.
   - Infer only what the evidence implies.
   - Check inferences for their consistency with each other.
   - Identify assumptions that lead you to your inferences.

3. All reasoning leads somewhere or has implications and consequences.
• Trace the implications and consequences that follow from your reasoning.
• Search for negative as well as positive implications.
• Consider all possible consequences.

If you would like to give a hint for a non-technical simulation, Gary Fowler (1996) suggests the following exercise (the suggestion has been tailored to our problem): Choose one student to be the Division Director and one to be the Internal Auditor. From a deck of playing cards, select an ace and two lower cards (to represent the fraud and non-fraud locations respectively). The Division Director holds the cards so only he/she can see their value. The Internal Auditor selects a card at random and places it aside without looking at its value. This card represents the CFO’s choice for the summary audit. The Division Director then reveals the value of one of the remaining cards which is not the ace (the fraud location). If the Internal Auditor switches the location of the summary audit by choosing the remaining card in the Division Director’s hand, he/she will only miss the fraud when the original choice was the ace. Therefore, he/she will find the fraud in the remaining cases, providing a probability of finding the fraud 2/3 of the time.

I have found this assignment to be a very effective classroom tool. I typically do not offer a hint to the entire class for the simulations. I may provide a hint or other guidance if students ask good questions privately – a good question being one that shows they have been working on the problem. I also like to give a fair amount of time between the assignment of the problem and the due date. I find this length of time increases the probability that more students will put in an adequate effort to reach a satisfactory conclusion.

Our profession is constantly in need of methods and materials that help us reach our education goals. We all have a duty to contribute and share with one another to improve the education possibilities for our students. The purpose of this paper is to contribute a teaching method that helps students improve their critical thinking, technology, and communication skills through solving a problem that involves constraints, uncertainty, and a real-world issue.

REFERENCES


